

Final Exam

December 12, 2005

This is a closed book examination. There is extra scratch paper available. Explain all answers completely – show your thoughts and work!

- 1) [10 pts] You have a spring with a small mass attached that has an oscillation frequency, f , when it is bounced vertically. If you cut your spring in 3 equal parts and use one of these shorter springs with the same mass the new oscillation frequency is

a) Less – $\frac{1}{\sqrt{3}} f$

b) Less – $\frac{1}{3} f$

c) The same – f

d) Greater – $\sqrt{3} f$

e) Greater – $3 f$

$\omega = 2\pi f \quad \omega = \left(\frac{k}{m}\right)^{1/2} \quad \text{so } f = \frac{1}{2\pi} \left(\frac{k}{m}\right)^{1/2}$

when spring is shortened k increases $k' = 3k \quad m' = m$

$f' = \frac{1}{2\pi} \left(\frac{k'}{m'}\right)^{1/2} = \frac{1}{2\pi} \left(\frac{3k}{m}\right)^{1/2} = \sqrt{3} \frac{1}{2\pi} \left(\frac{k}{m}\right)^{1/2} = \sqrt{3} f$

- 2) [10 pts] You throw a light ball into the air at an angle of θ to the horizontal. After the ball leaves your hand it travels a vertical distance y . You toss another ball that has 9 times the mass of the first ball into the air again exactly as you did before except you throw this ball with 3 times the initial velocity. The resulting vertical distance is

a) Less – $\frac{1}{3} y$

b) Less – $\frac{1}{9} y$

c) The same – y

d) Greater – $3y$

e) Greater – $9y$

$mgh = \frac{1}{2} m v_y^2$ a height is independent of mass

$h = \frac{v_y^2}{2g} \quad v_y' = 3v_y \quad \text{so } h' = \frac{(3v_y)^2}{2g} = 9 \frac{v_y^2}{2g} = 9h$

- 3) [10 pts] You have a ring and a solid disk that both have the same radius. However, the disk has four times the mass of the ring. If these objects were to roll without slipping down a 7° ramp which one would reach the bottom first?

a) The ring $I_1 = m_1 r^2$

b) The disk $I_2 = \frac{1}{2} m_2 r^2$ smaller coefficient means it is faster – (see work)

c) They would reach the bottom at the same time

d) They would not reach the bottom, the ramp needs to be steeper

- 4) [10 pts] You are pushing a box horizontally across the floor at a constant velocity of 2 m/s. You get tired and start pushing with half the original force. If the box travels 2 meters after you get tired, what is the kinetic coefficient of friction? $\mu_k = 0.2$

- 5) [10 pts] What is the escape velocity for a 3.14×10^4 kg rocket launched from earth?

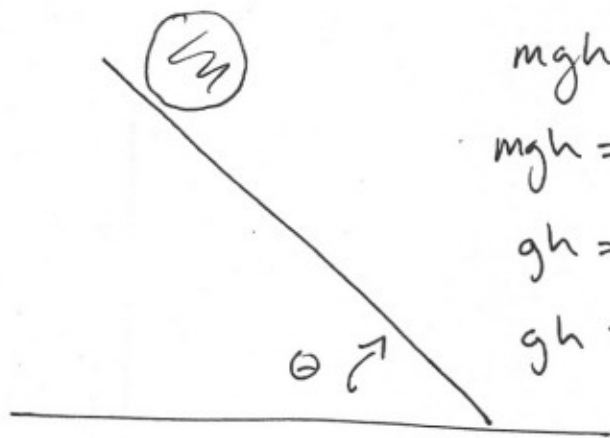
(independent of mass) $v_{esc} = 11 \text{ km/sec}$

- 6) [10 pts] An amusement park is testing the failure modes of their new roller coaster. They release an empty 3-car chain from the top of a 50 m tall ramp. At the bottom of the ramp is a horizontal section of track 10 m long. In the middle of this section is a 2-car chain. Following the flat section is a ramp that carries the cars to the top of a 20 m tall hill. If the first 3 empty cars were to stick to the bottom cars would the resulting 5-car chain make it over the second ramp? Assume negligible friction.

- 7) [10 pts] If $\Sigma F = 0$ on an object, can it stop spinning? If $\Sigma \tau = 0$ on an object, can it accelerate? Can an object be moving if $\Sigma F = 0$ and $\Sigma \tau = 0$? Explain.

FINAL EXAM

#3 In general



$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}A m r^2 \omega^2$$

$$gh = \frac{1}{2}v^2 + \frac{1}{2}A(r^2\omega^2)$$

$$gh = \frac{1}{2}v^2 + \frac{1}{2}A v^2 = \frac{(1+A)}{2}v^2$$

$$v = r\omega$$

$$I = A m r^2$$

A is a coefficient dependent on object geometry

$$v^2 = \frac{2gh}{(1+A)}$$

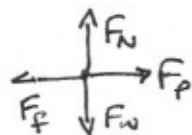
smaller A means greater velocity —

- Note: result is independent of mass and radius of object

#4



$$\sum F = ma$$



① Initially $\Delta v = 0$ $v_i = \frac{2m}{s}$ since

$$F_p = F_f = 0 \quad \text{so} \quad F_p = F_f = \mu_k F_N = \mu_k F_w$$

$$F_N - F_w = 0$$

$$d = 2m$$

$$v_0 = 2m/s$$

$$a = -\frac{1}{2}\mu_k g$$

$$v = 0m/s \text{ stops}$$

② $\frac{1}{2}F_p - F_f = ma$ $F_N - F_w = 0$ $F_w = mg$

$$\frac{1}{2}(\mu_k F_w) - \mu_k F_w = ma \quad \text{so} \quad a = -\frac{1}{2}\mu_k g$$

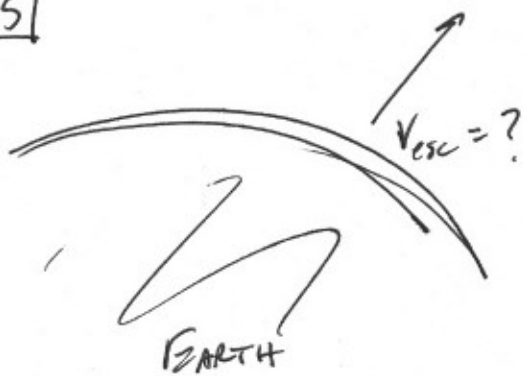
Using kinematics

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \quad \text{and} \quad v = v_0 + at \quad \text{we find}$$

$$v^2 - v_0^2 = 2ad \quad \text{so} \quad \frac{v_0^2}{2d} = \frac{\mu_k g}{2} \quad \text{so} \quad \mu_k = \frac{v_0^2}{dg}$$

$$\mu_k = 0.2$$

#5



$$E_i = E_f$$

$$\frac{1}{2}mv^2 + \frac{-GM_E m}{R_E} = 0 \quad \text{at } R = \infty \quad P_{Ea} = 0$$

$$\frac{1}{2}mv^2 = \frac{GM_E m}{R_E}$$

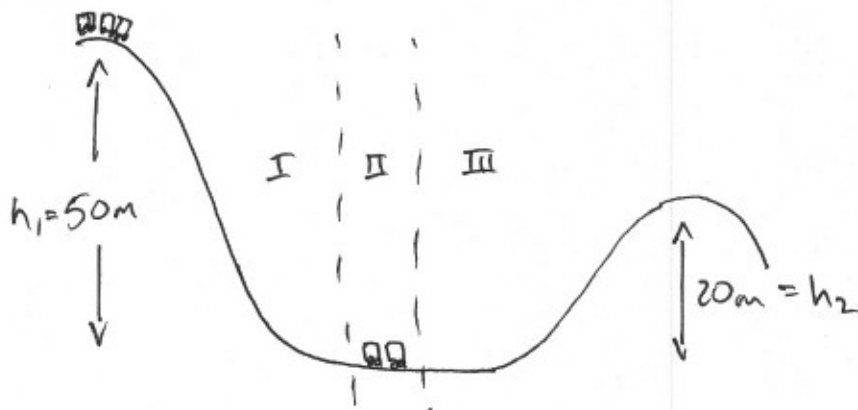
$$v^2 = \frac{2GM_E}{R_E}$$

$$v = \left(\frac{2GM_E}{R_E} \right)^{1/2} = \left(2 \cdot 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \cdot \frac{5.98 \times 10^{24} \text{kg}}{6370 \times 10^3 \text{m}} \right)^{1/2} = 1.119 \times 10^4 \text{ m/s}$$

$$v_{\text{escape}} = 11,190 \text{ m/s} \quad \text{or} \quad 25,179 \text{ mph}$$

Note: units work ✓ v_{escape} is independent of mass
(However, energy required is dependent on mass)

#6



I: Energy Conservation $\frac{1}{2}mv_1^2 = mgh_1 \quad v_1 = (2gh_1)^{1/2} \quad \text{or} \quad v_1^2 = 2gh_1$

II: Momentum Conservation $(3m)v_1 = (5m)v_2 \quad v_2 = \frac{3}{5}v_1$

III: Energy Conservation $\frac{1}{2}m'v_2^2 = m'gh_2 \quad h_2 = \frac{v_2^2}{2g}$

$$h_2 = \frac{\left(\frac{3}{5}v_1\right)^2}{2g} = \left(\frac{3}{5}\right)^2 \frac{2gh_1}{2g} = \left(\frac{3}{5}\right)^2 h_1$$

units ✓

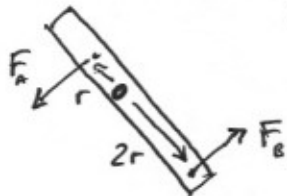
$$h_2 = \frac{9}{25} \cdot 50 \text{m} = 18 \text{m}$$

Scars do not make it over the 20m tall hill

#7

$$\sum \vec{F} = m\vec{a} \quad \text{and} \quad \sum \vec{\tau} = I\vec{\alpha}$$

- (a) IF $\sum \vec{F} = 0$ an object could have $\sum \vec{\tau} \neq 0$ so
it could have a non-zero angular accel.
and either speed up or slow down
its rotation

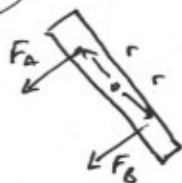


Example:

$F_A = F_B$ but τ_A and τ_B want to rotate
in the same direction

- (b) IF $\sum \vec{\tau} = 0$ an object could have $\sum \vec{F} \neq 0$
so it would have a net acceleration

Example:



$F_A = F_B$ but $\tau_A = -\tau_B$ object moves

- (c) IF $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$ then acceleration and
angular acceleration are zero

$$a = 0 \rightarrow \Delta v = 0$$

$$\alpha = 0 \rightarrow \Delta \omega = 0$$

This means an object
can be moving but will
not speed-up or slow-down.

- constant velocity
- constant ~~rotational speed~~
angular velocity
(rotation speed)